

Pearson Edexcel Level 3 GCE

Practice Paper 5

Time: 1 hour 30 minutes

Paper Reference **9FM0/4D**

Further Mathematics

Advanced

Paper 4D: Decision Mathematics 2

You must have:
calculator

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 84. There are 7 questions.
- The marks for each question are shown in brackets
 - use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

1. A sequence satisfies the recurrence relation $u_n = 5u_{n-1} - 4u_{n-2}$, with $u_0 = 20$ and $u_1 = 19$.

(a) Solve the recurrence relation to find a closed form for u_n . **(5)**

(b) Show that the sequence is decreasing and that $u_n < 0$ for all $n \geq 3$. **(3)**

(Total for Question 1 is 8 marks)

2. Emma and Freddie play a zero-sum game. This game is represented by the following pay-off matrix for Emma.

$$\begin{pmatrix} -4 & -1 & 3 \\ 2 & 1 & -2 \end{pmatrix}$$

(a) Show that there is no stable solution. **(3)**

(b) Find the best strategy for Emma and the value of the game to her. **(8)**

(c) Write down the value of the game to Freddie and his pay-off matrix. **(3)**

(Total for Question 2 is 14 marks)

3. Five tour guides work at a stately home. They talk to groups of tourists about five particularly significant rooms. Each tour guide will be stationed in a particular room for the day but may change rooms the next day. The tourists will listen to each talk before moving on to the next room. Once they have listened to all five talks they will head off to the gift shop.

The table shows the average length of each tour guide's talk in each room.

	Grand Hall	Dining Room	Gallery	Bedroom	Kitchen
Alf	8	19	11	14	12
Betty	12	17	14	18	20
Charlie	10	22	18	14	19
Donna	9	15	16	15	21
Eve	14	23	20	20	19

A tourist arrives at the stately home.

- (a) Use the Hungarian algorithm, reducing rows first, to find the shortest time that the tour could take. You should state each optimal allocation and its length and show the state of the table at each stage. (7)
- (b) Adapt the table and reapply the Hungarian algorithm, reducing rows first, to find the longest time that the tour could take. You should state each optimal allocation and its duration and show the state of the table at each stage. (8)

(Total for Question 3 is 15 marks)

4. A group of students and teachers from a performing arts college are attending the Glasenburgh drama festival. All of the group want to see an innovative modern production of the play 'The Decision is Final'. Unfortunately there are not enough seats left for them all to see the same performance.

There are three performances of the play, 1, 2, and 3. There are two types of ticket, Adult and Student. Student tickets will be purchased for the students and Adult tickets for the teachers.

The table below shows the price of tickets for each performance of the play.

	Adult	Student
Performance 1	£5.00	£4.50
Performance 2	£4.20	£3.80
Performance 3	£4.60	£4.00

There are 18 teachers and 200 students requiring tickets.

There are 94, 65 and 80 seats available for performances 1, 2, and 3 respectively.

- (a) Complete the first table in the answer book. (2)
- (b) Explain why a dummy column was added to the table in the answer book. (1)
- (c) Use the north-west corner method to obtain a possible solution. (1)
- (d) Taking the most negative improvement index to indicate the entering square, use the stepping stone method **once** to obtain an improved solution. You must make your shadow costs and improvement indices clear. (6)

After a further iteration the table becomes:

	Adult	Student	Dummy
Performance		73	21

1			
Performance 2	18	47	
Performance 3		80	

(e) Demonstrate that this solution gives the minimum cost, and find its value.

(6)

(Total for Question 4 is 16 marks)

5.

Figure 1

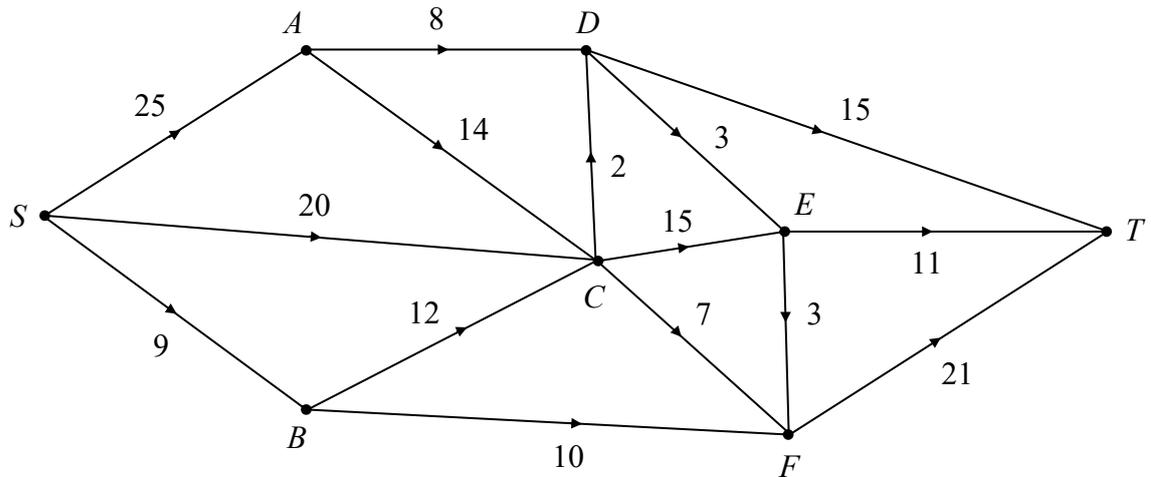


Figure 4 shows a capacitated directed network. The number on each arc is its capacity.

(a) State the maximum flow along

(i) *SADT*,

(ii) *SCET*,

(iii) *SBFT*.

(2)

(b) Show these maximum flows on Diagram 1 in the answer book.

(1)

Take your answer to part (b) as the initial flow pattern.

(c) (i) Use the labelling procedure to find a maximum flow from *S* to *T*. Your working should be shown on Diagram 2 in the answer book. List each flow-augmenting route you use, together with its flow.

(5)

(ii) Draw your final flow pattern on Diagram 3 in the answer book.

(2)

(iii) Prove that your flow is maximal.

(3)

(d) Give an example of a practical situation that could have been modelled by the original network.

(1)

(Total for Question 5 is 14 marks)

6. The management team of a company are considering taking on a project. If the project is very successful, then it can earn £75 000 for the company. If it is moderately successful, it can earn £45 000 for the company. If it is unsuccessful, then the company can lose £30 000.

The team estimate that the probability of a very successful outcome is 20% and the probability of a moderately successful outcome is 50%.

The company can chose to hire a business consultant at a cost of £ x . The consultant can provide advice that will increase the probability of a very successful outcome to 35% and the probability of a moderately successful outcome to 55%.

- (a) Draw a decision tree to model the company's choices and outcomes. (6)
- (b) Write, in terms of x , the EMV of the project if the company decides to hire the consultant. (2)
- (c) Determine the maximum amount that the company should pay to hire the consultant. (2)

(Total for Question 6 is 10 marks)

7. Four workers, A, B, C and D, are to be assigned to four tasks, 1, 2, 3 and 4. Each worker must be assigned to just one task and each task must be done by just one worker.

Worker C cannot do task 4 and worker D cannot do task 1.

The cost of assigning each worker to each task is shown in the table below.
The total cost is to be minimised.

	1	2	3	4
A	29	15	32	30
B	34	26	40	32
C	28	27	35	–
D	–	21	33	31

Formulate the above situation as a linear programming problem. You must define your decision variables and make the objective function and constraints clear.

(Total for Question 7 is 7 marks)

TOTAL FOR DECISION MATHEMATICS 2 IS 84 MARKS

Looking for answers or mark schemes? The source of these questions is either the Pearson D2 textbook or past papers which can be found on the Emporium website.

1. p234, q8
2. D2 June 2004, q4
3. p68, q3
4. D2 June 2007, q4
5. D1 January 2005, q6
6. p269, q27
7. D2(R) June 2014, q6